

Scilab Textbook Companion for
Discrete Mathematics
by S. Lipschutz, M. Lipson And V. H. Patil¹

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Book Description

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Scilab numbering policy used in this document and the relation to the above book.

Exa Example (Solved example)

Eqn Equation (Particular equation of the above book)

AP Appendix to Example(Scilab Code that is an Appednix to a particular Example of the above book)

For example, Exa 3.51 means solved example 3.51 of this book. Sec 2.3 means a scilab code whose theory is explained in Section 2.3 of the book.

Contents

List of Scilab Codes	4
1 Set Theory	5
3 Functions and Algorithms	9
5 Vectors and Matrices	12
6 Counting	16
7 Probability Theory	23
8 Graph Theory	40
9 Directed graphs	43
11 Properties of the integers	45
12 Algebraic Systems	50
15 Boolean Algebra	52
16 Recurrence relations	54

List of Scilab Codes

Exa 1.8	inclusion exclusion principle	5
Exa 1.9	Inclusion exclusion principle	6
Exa 1.13	Power sets	7
Exa 1.14	Power sets	7
Exa 1.18	Mathematical induction	7
Exa 3.8	Recursively defined functions	9
Exa 3.9	Cardinality	9
Exa 3.10	Polynomial evaluation	10
Exa 3.11	Greatest Common Divisor	10
Exa 5.2	Vector operations	12
Exa 5.3	Column vectors	12
Exa 5.5	Matrix addition and Scalar multiplication	13
Exa 5.6	Matrix multiplication	13
Exa 5.7	Matrix multiplication	13
Exa 5.8	Algebra of square matrices	14
Exa 5.9	Invertible matrices	14
Exa 5.10	Determinants	14
Exa 5.13	Matrix solution of a system of linear equations	15
Exa 5.14	Inverse of a square matrix	15
Exa 6.1	Sum rule principle	16
Exa 6.2	Product rule principle	17
Exa 6.3	Factorial notation	17
Exa 6.4	Binomial coefficients	18
Exa 6.5	Permutations	18
Exa 6.6	Permutations with repetitions	19
Exa 6.7	Combinations	19
Exa 6.8	Combinations	20
Exa 6.9	Combinations with repetitions	20

Exa 6.14	Ordered partitions	21
Exa 6.15	Unordered partitions	21
Exa 6.16	Inclusion exclusion principle revisited	22
Exa 7.1	Sample space and events	23
Exa 7.2	Finite probability spaces	24
Exa 7.3	Equiprobable spaces	25
Exa 7.4	Addition principle	25
Exa 7.5	Conditional probability	26
Exa 7.6	Multiplication theorem for conditional probability	28
Exa 7.7	Independent events	29
Exa 7.8	Independent events	30
Exa 7.9	Independent repeated trials	30
Exa 7.10	Repeated trials with two outcomes	31
Exa 7.12	Random variables	32
Exa 7.13	Probability distribution of a random variable	33
Exa 7.14	Probability distribution of a random variable	34
Exa 7.15	Expectation of a random variable	35
Exa 7.16	Variance and standard deviation of a random variable	36
Exa 7.17	Binomial diatribution	37
Exa 7.18	Chebyshev inequality	38
Exa 7.19	Sample mean and Law of large numbers	38
Exa 8.1	Paths and connectivity	40
Exa 8.2	Minimum spanning tree	41
Exa 9.6	Adjacency matrix	43
Exa 9.8	Path matrix	43
Exa 11.2	Division algorithm	45
Exa 11.4	Primes	45
Exa 11.5	Greatest Common Divisor	46
Exa 11.6	Euclidean algorithm	46
Exa 11.9	Fundamental theorem of Arithmetic	47
Exa 11.12	Congruence relation	48
Exa 11.19	Linear congruence equation	48
Exa 12.4	Properties of operations	50
Exa 12.17	Roots of polynomial	50
Exa 12.18	Roots of polynomial	51
Exa 15.1	Basic definitions in boolean algebra	52
Exa 15.2	Boolean algebra as lattices	53

Exa 16.14	Linear homogenous recurrence relations with constant coefficients	54
Exa 16.15	Solving linear homogenous recurrence relations with constant coefficients	55
Exa 16.16	Solving linear homogenous recurrence relations with constant coefficients	56
Exa 16.17	Solving linear homogenous recurrence relations with constant coefficients	57
Exa 16.18	Solving linear homogenous recurrence relations with constant coefficients	58
Exa 16.19	Solving general homogenous linear recurrence relations	59

Chapter 1

Set Theory

Scilab code Exa 1.8 inclusion exclusion principle

```
1 disp('To find: number of mathematics students taking
      atleast one of the languages French(F), German(G)
      and Russian(R)')
2 F=65; //number of students studying French
3 G=45; // number of students studying German
4 R=42; //number of students studying Russian
5 FandG=20; //number of students studying French and
      German
6 FandR=25; //number of students studying French and
      Russian
7 GandR=15; //number of students studying German and
      Russian
8 FandGandR=8; //number of students studying French,
      German and Russian
9 //By inclusion-exclusion principle
10 ForGorR=F+G+R-FandG-FandR-GandR+FandGandR;
11 disp(ForGorR, 'the number of students studying
      atleast one of the languages :')
```

Scilab code Exa 1.9 Inclusion exclusion principle

```
1 disp('In a college ,120 mathematics students can opt
      for either French(F),German(G) or Russian(R)')
2 n=120; //total number of students
3 F=65; //number of students studying French
4 G=45; //number of students studying German
5 R=42; //number of students studying Russian
6 FandG=20; //number of students studying French and
      German
7 FandR=25; //number of students studying French and
      Russian
8 GandR=15; //number of students studying German and
      Russian
9 FandGandR=8; //number of students studying French,
      German and Russian
10 disp('using inclusion-exclusion principle:')
11 ForGorR=F+G+R-FandG-FandR-GandR+FandGandR;
12 disp(ForGorR,'number of students studying French or
      German or Russian')
13 FGnR=FandG-FandGandR;
14 disp(FGnR,'number of students studying French and
      German but not Russian')
15 FRnG=FandR-FandGandR ;
16 disp(FRnG,'number of students studying French and
      Russian but not German')
17 GRnF=GandR-FandGandR ;
18 disp(GRnF,'number of students studying German and
      Russian but not French')
19 OF=F-FGnR-FandGandR-FRnG ;
20 disp(OF,'number of students studying Only French')
21 OG=G-FGnR-FandGandR-GRnF;
22 disp(OG,'number of students studying Only German')
23 OR=R-FRnG-FandGandR-GRnF;
24 disp(OR,'number of students studying Only Russian')
25 k=n-ForGorR;
26 disp(k,'number of students not studying any of the
      languages')
```

Scilab code Exa 1.13 Power sets

```
1 x=10; //number of members of set X
2 P=2^x //number of members of the power set of X
3 q=P-1; //x itself is not the proper subset.Hence it
   isn't counted
4 disp(q, 'number of members of powerset P which are
   proper subsets of x are:')
```

Scilab code Exa 1.14 Power sets

```
1 A=[1,2,3,4,5]; //eatables for salad preparation 1=
   onion,2=tomato,3=carrot,4=cabbage,5=cucumber
2 p=length(A); //total number of eatables available
3 n=2^p-1; //no salad can be made without atleast one
   of the eatables.Hence null set isn't counted
4 disp(n, 'number of different salads that can be
   prepared using the given eatables')
```

Scilab code Exa 1.18 Mathematical induction

```
1 U1=1; //given
2 U2=5; //given
3 P=[];
4 for i=1:2
5 P(i)=3^i-2^i;
6 disp(P(i))
7 end
8 disp('P(1)=U(1) and P(2)=U(2)');
```

9 `disp('hence $U_n=3^n-2^n$ for all n belonging to \mathbb{N} ');`

Chapter 3

Functions and Algorithms

Scilab code Exa 3.8 Recursively defined functions

```
1 function [k]=fact(a)
2 k=-1;
3 if(a<0|a>200)
4 disp(" Invalid");
5 break;
6 else
7 if(a==1|a==0)
8 k=1;
9 else
10 k=a*fact(a-1);
11 end
12 end
13 endfunction
14 a=4;
15 p=fact(a);
16 disp(p,'the value of 4! is')
```

Scilab code Exa 3.9 Cardinality

```

1 x=1;
2 y=2;
3 z=3;
4 A=[x,y,z];
5 disp('cardinality of set A is:')
6 length(A)
7 B=[1,3,5,7,9]
8 disp('cardinality of set B is:')
9 length(B)
10
11 // 3.9 (b)
12 disp('the set E has the following elements)
13 E=[2,4,6 %inf] //set E is the set of all positive
    even numbers and N is the set of all natural
    numbers
14 disp('function f:N to E is defined.So,E has the same
    cardinality as N')
15 disp('set E is countably infinite:')
16 for x=2:2:%inf
17 y=2*x;
18 disp(y)
19 end

```

Scilab code Exa 3.10 Polynomial evaluation

```

1 x = poly(0, 'x');
2 p = 2*x^3-7*x^2+4*x-15;
3 disp(p,'the polynomial is')
4 k=horner(p,5);
5 disp(k,'value of the polynomial at x=5 is')

```

Scilab code Exa 3.11 Greatest Common Divisor

```
1 V=int32([258,60]);
2 thegcd=gcd(V);
3 disp(thegcd,'the gcd of the two numbers 258 and 60
   is')
```

Chapter 5

Vectors and Matrices

Scilab code Exa 5.2 Vector operations

```
1 u=[2,3,-4];
2 v=[1,-5,8];
3 u+v
4 5*u
5 -v
6 2*u-3*v
7 u.*v;
8 k=sum(u.*v);
9 disp(k,'dot product of the two vectors')
10 l=norm(u);
11 disp(l,'norm or length of the vector u')
```

Scilab code Exa 5.3 Column vectors

```
1 u=[5,3,-4]';
2 v=[3,-1,-2]';
3 2*u-3*v
4 k=sum(u.*v);
```

```
5 disp(k, 'The dot product of the two vectors u and v
   is: ')
6 l=norm(u);
7 disp(l, 'The length or norm of the vector u is:')
```

Scilab code Exa 5.5 Matrix addition and Scalar multiplication

```
1 A=[1, -2, 3; 0, 4, 5];
2 B=[4, 6, 8; 1, -3, -7];
3 k=A+B;
4 disp(k, 'The addition of the two matrices A and B is:
   ')
5 m=3*A;
6 disp(m, 'The multiplication of a vector with a scalar
   is: ')
7 p=2*A-3*B
```

Scilab code Exa 5.6 Matrix multiplication

```
1 a=[7, -4, 5];
2 b=[3, 2, -1]';
3 k=a*b;
4 disp(k, 'product of a and b is;')
5 p=[6, -1, 8, 3];
6 q=[4, -9, -2, 5]';
7 l=p*q;
8 disp(l, 'product of p and q is:')
```

Scilab code Exa 5.7 Matrix multiplication


```

1 A=[1 3;2 -1];
2 B=[2 0 -4;5 -2 6];
3 A*B
4 A=[1 2;3 4]
5 B=[5 6;0 -2];
6 A*B
7 B*A
8 disp('matrix mulitplication is not commutative since
      AB may not be equal to BA')

```

Scilab code Exa 5.8 Algebra of square matrices

```

1 A=[1 2;3 -4];
2 A2=A*A //multiplying A by itself
3 A3=A2*A
4 f=2*A2-3*A+5;
5 disp(f, 'for the function  $f(x)=2x^2-3x+5$ ,  $f(A)$  is : ')
6 g=A2+3*A-10;
7 disp(g, 'for the function  $g(x)=x^2+3x-10$ ,  $g(A)$  is ')

```

Scilab code Exa 5.9 Invertible matrices

```

1 A=[1 0 2;2 -1 3;4 1 8];
2 B=[-11 2 2;-4 0 1;6 -1 -1];
3 A*B
4 disp('since A*B is identity matrix ,A and B are
      invertible and inverse of each other ')

```

Scilab code Exa 5.10 Determinants

```

1 A=[5 4;2 3];
2 det(A);
3 disp(det(A), 'determinant of A')
4 B=[2 1;-4 6]
5 det(B);
6 disp(det(B), 'determinant of B')
7 C=[2 1 3;4 6 -1;5 1 0]
8 disp(det(C), 'determinant of C')

```

Scilab code Exa 5.13 Matrix solution of a system of linear equations

```

1 A=[1 2 1;2 5 -1;3 -2 -1]; //left hand side of
   the system of equations
2 B=[3 -4 5]'; //right hand side or
   the constants in the equations
3 X=[];
4 X=A\B ; //unique solution for the system of
   equations
5 x=X(1)
6 y=X(2)
7 z=X(3)

```

Scilab code Exa 5.14 Inverse of a square matrix

```

1 A=[1 0 2;2 -1 3;4 1 8];
2 P=rref([A, eye(3,3)]);
3 disp(P, 'canonical form of matrix A :')
4 disp('left side of the matrix P is the identity
   matrix so the right side is the inverse of A')
5 inverseA=P(:,4:6)

```

Chapter 6

Counting

Scilab code Exa 6.1 Sum rule principle

```
1 M=8;    //number of male professors teaching
        calculus
2 F=5;    //number of female professors teaching
        calculus
3 T=M+F ;
4 disp(T, 'number of ways a student can choose a
        calculus professor ')
5
6 E=[2,3,5,7]; //event of choosing a prime number
        less than 10
7 F=[2,4,6,8]; //event of choosing an even number
        less than 10
8 G=intersect(E,F); //event of getting an even and
        prime number
9 H=length(E)+length(F)-length(G);
10 disp(H, 'event of getting an even or a prime number')
11
12 E=[11,13,17,19]; //event of choosing a prime number
        between 10 and 20
13 F=[12,14,16,18]; //event of choosing an even number
        between 10 and 20
```

```

14 G=union(E,F);    //event of choosing a number which
    is prime or even
15 k=length(G);
16 disp(k,'number of ways of choosing a number which is
    prime or even')

```

Scilab code Exa 6.2 Product rule principle

```

1  disp('a license plate contains two letters followed
    by three digits where first digit can not be zero
    ')
2  n=26; //number of english letters
3  n*n; //number of ways of choosing two letters in
    the license plate
4  p=10; //number of digits (0-9)
5  (p-1)*p*p; //number of ways to select the three
    digits with the first digit not being zero
6  k=n*n*(p-1)*p*p;
7  disp(k,'total number of license plates that can be
    printed')
8
9  disp('a president ,a secretary and a treasurer has
    to be elected in an orga-nisation of 26 members.
    No person is elected to more than one postion')
10 t=26; //total number of members in the organisation
11 j=t*(t-1)*(t-2);
12 disp(j,'number of ways to elect the three officers (
    president ,secretary ,treasurer ')

```

Scilab code Exa 6.3 Factorial notation

```

1  disp('To find: factorial of a 6')
2  facto2=2*1;

```

```

3 facto3=3*facto2
4 facto4=3*facto3
5 facto4=4*facto3
6 facto5=5*facto4
7 facto6=6*facto5
8
9 k=8*7*factorial(6)/factorial(6);
10 disp(k, 'value of 8!/6! is:')
11 j=12*11*10*factorial(9)/factorial(9);
12 disp(j, 'value of 12!/9! is:')

```

Scilab code Exa 6.4 Binomial coefficients

```

1 function [k]=func1(n,r) //calculating binomial
   coefficient
2 k=factorial(n)/(factorial(r)*factorial(n-r));
3 endfunction
4 func1(8,2)
5 func1(9,4)
6 func1(12,5)
7 func1(10,3)
8 func1(13,1)
9
10 p = factorial(10)/(factorial(10-7)*factorial(7));
   //calculating 10C7
11 q= factorial(10)/(factorial(10-3)*factorial(3)) //
   calculating 10C3
12 disp(p, 'value of 10C7 is ')
13 //10-7=3 so 10C7 can also be computed as 10C3
14 //both p and q have same values but second method
   saves time and space

```

Scilab code Exa 6.5 Permutations

```

1 disp('finding the number of three-letter words using
      only the given six letters (A,B,C,D,E,F) without
      repetition ')
2 n=6; //total number of letters
3 l1=n; //number of ways in which first letter of the
      word can be chosen
4 l2=n-1; //number of ways in which second letter of
      the word can be chosen
5 l3= n-2; //number of ways in which third letter can
      be chosen
6 k=l1*l2*l3;
7 disp(k, 'number of three-letter words possible ')

```

Scilab code Exa 6.6 Permutations with repetitions

```

1 function [k]= funct1(n,p,q)
2 k= factorial(n)/(factorial(p)*factorial(q));
3 endfunction
4 k=funct1(7,3,2) //in "BENZENE" three letters are
      alike(the three Es) and two are alike (the two Ns
      )
5 disp(k, 'The number of seven-letter words that can be
      formed using letters of the word BENZENE')
6
7 disp('a set of 4 indistinguishable red coloured
      flags , 3 indistinguishable white flags and a blue
      flag is given ')
8 j=funct1(8,4,3);
9 disp(j, 'number of different signals ,each consisting
      of eight flags ')

```

Scilab code Exa 6.7 Combinations

```

1 disp('four objects are given (a,b,c,d) and three are
      taken at a time')
2 combinations = factorial(4)/(factorial(4-3)*
      factorial(3));
3 disp(combinations,'number of combinations of the
      four objects given')
4 k=factorial(3); //number of permutations of objects
      in a combination
5 permutations = combinations*k;
6 disp(permutations,'total number of permutations for
      the problem')

```

Scilab code Exa 6.8 Combinations

```

1 function [k]= myfunc(n,r)
2 k=factorial(n)/(factorial(n-r)*factorial(r));
3 endfunction
4 k=myfunc(8,3);
5 disp(k,'the number of committees of three that can
      be formed out of eight people')
6
7 cows=myfunc(6,3) //number of ways that a farmer can
      choose 3 cows out of 6 cows
8 pigs=myfunc(5,2) //number of ways that a farmer can
      choose 2 pigs out of 5 pigs
9 hens=myfunc(8,4) //number of ways that a farmer can
      choose 4 hens out of 8 hens
10 p=cows*pigs*hens;
11 disp(p,'total number of ways that a farmer can
      choose all these animals')

```

Scilab code Exa 6.9 Combinations with repetitions

```

1 //each solution to the equation can be viewed as a
  combination of objects
2 r=18; //number of objects
3 M=3; //kinds of object
4 m=factorial(r+(M-1))/(factorial(r+(M-1)-(M-1))*
  factorial(M-1));
5 disp(m, 'number of non negative integer solutions of
  the given equation x+y+z=18')

```

Scilab code Exa 6.14 Ordered partitions

```

1 c1=3; //number of toys that the youngest child
  should get
2 c2=2; //number of toys that the third child should
  get
3 c3=2; //number of toys that the second child should
  get
4 c4=2; //number of toys that the eldest son should
  get
5 m=factorial(9)/(factorial(3)*factorial(2)*factorial
  (2)*factorial(2));
6 disp(m, 'number of ways nine toys can be divided
  between four children with the youngest son
  getting 3 toys and others getting 2 each')

```

Scilab code Exa 6.15 Unordered partitions

```

1 p=12; //total number of students
2 t=3; //number of teams or partition
3 disp('each partition of the students can be arranged
  in 3! ways as an ordered partition')
4 r=factorial(12)/(factorial(4)*factorial(4)*factorial
  (4)) //number of ordered partitions

```



```
5 m=r/factorial(t); //number of unordered partitions
6 disp(m, 'number of ways that 12 students can be
    partitioned into three teams so that each team
    consists of 4 students')
```

Scilab code Exa 6.16 Inclusion exclusion principle revisited

```
1 U=1000; //number of elements in the set of positive
    integers not exceeding 1000
2 A=U/3; //number of elements in the subset of
    integers divisible by 3
3 B=U/5; //number of elements in the subset of
    integers divisible by 5
4 C=U/7; //number of elements in the subset of
    integers divisible by 7
5 AandB=floor(U/(3*5)) //number of elements in the
    subset containing numbers divisible by both 3 and
    5
6 AandC=floor(U/(3*7)) //number of elements in the
    subset containing numbers divisible by both 3 and
    7
7 BandC=floor(U/(5*7)) //number of elements in the
    subset containing numbers divisible by both 5 and
    7
8 AandBandC=floor(U/(3*5*7)) //number of elements in
    the subset containing numbers divisible by 3,5
    and 7
9 s=U-(A+B+C)+(AandB+AandC+BandC)-(AandBandC); // By
    inclusion-exclusion principle
10 S=round(s);
11 disp(S, 'The number of integers in the set U, which
    are not divisible by 3,5 and 7 is')
```

Chapter 7

Probability Theory

Scilab code Exa 7.1 Sample space and events

```
1 S=[1,2,3,4,5,6];           //sample space for the
    rolling of a die
2 A=[2,4,6];                 //event that an even number
    occurs
3 B=[1,3,5];                 //event that an odd number
    occurs
4 C=[2,3,5];                 //event that a prime number
    occurs
5 disp(union(A,C), 'sample space for the event that an
    even or a prime number occurs')
6 disp(intersect(B,C), 'sample space for the event that
    an odd prime number occurs')
7 disp(setdiff(S,C), 'sample space for the event that a
    prime number does not occur') //It is the
    complement of the set C.
8 intersect(A,B)             //It is a null set or null vector
    since there can't occur an even and an odd
    number simultaneously
9
10 H=0;                       //"head" face of a coin
11 T=1;                       //"tail" face of a coin
```

```

12 S=["000","001","010","011","100","101","110","111"]
    ; //sample space for the toss of a coin three
    times
13 A=["000","001","100"]; //event that two more or
    more heads appear consecutively
14 B=["000","111"]; //event that all tosses
    are the same
15 disp(intersect(A,B),'sample space for the event in
    which only heads appear')
16
17 disp('Experiment:tossing a coin until a head appears
    and then counting the number of times the coin
    is tossed')
18 S=[1,2,3,4,5,%inf] //The sample space has
    infinite elements in it
19 disp("Since every positive integer is an element of
    S,the sample space is infinite")

```

Scilab code Exa 7.2 Finite probability spaces

```

1 disp('Experiment:three coins are tossed and the
    number of heads are observed')
2 S=[0,1,2,3]; //the sample space for the experiment
    where 0 implies no heads,1 implies only one head
    out of the three coins and so on
3 disp("the probability space is as follows ")
4 P0=1/8; //probability of getting no head on any of
    the coins i.e TTT
5 P1=3/8; //probability of getting only one head on
    any of the coins, out of the three coins i.e HTT,
    THT,TTH
6 P2=3/8; //probability of getting two heads, out of
    the three coins i.e THH,HTH,HHT
7 P3=1/8; //probability of getting all the three
    heads i.e HHH

```

```

8 disp("A is the event that atleast one head appears
      and B is the event that all heads or all tails
      appear ")
9 A=[1,2,3]; // HHH,HHT,HTH,HTT,THH,THT,TTH
10 B=[0,3]; //HHH,TTT
11 PA=P1+P2+P3;
12 disp(PA,'probability of occurrence of event A')
13 PB=P0+P3;
14 disp(PB,'probability of occurrence of event B')

```

Scilab code Exa 7.3 Equiprobable spaces

```

1 disp("Experiment: a card is selected from a deck of
      52 cards ")
2 disp("A is the event of the selected card being a
      spade ")
3 disp("B is the event of the selected card being a
      face card ")
4 t=52 ; //the total number of cards
5 s=13; //number of spades
6 PA= s/t;
7 disp(PA,'probability of selecting a spade')
8 f=12; //number of face cards(jack,queen,king)
9 PB=f/t;
10 disp(PB,'probability of selecting a face card')
11 sf=3; //number of spade face cards
12 Psf=sf/t;
13 disp(Psf,"probability of selecting a spade face card
      is:")

```

Scilab code Exa 7.4 Addition principle

```

1 disp("Experiment: selection of a student out of 100
      students ")
2 M=30; //no of students taking mathematics
3 C=20; //no of students taking chemistry
4 T=100; //total no. of students
5 PM = M/T //probability of the selected student
      taking mathematics
6 PC = C/T //probability of the selected student
      taking chemistry
7 MnC=10; //no of students taking mathematics and
      chemistry
8 PMnC = MnC/T //probability of the selected student
      taking mathematics and chemistry both
9 PMorC = PM+PC-PMnC;
10 disp(PMorC, 'probability of the selected student
      taking mathematics or chemistry ')

```

Scilab code Exa 7.5 Conditional probability

```

1
2
3 //EXAMPLE 7.5 (a)
4
5
6 disp(" Experiment: A die is tossed and the outcomes
      are observed");
7
8
9 disp("To find: probability (PM) of an event that
      one of the dice is 2 if the sum is 6");
10
11
12 E=[" (1,5) ", " (2,4) ", " (3,3) ", " (4,2) ", " (5,1) "] //
      event that the sum of the two numbers on the two
      dice is 6

```

```

13
14
15 A=["(2,1)","(2,2)","(2,3)","(2,4)","(2,5)","(2,6)","
    (1,2)","(3,2)","(4,2)","(5,2)","(6,2)"] //event
    that 2 appears on atleast one die
16
17
18 B= intersect(A,E) //possible combination of
    numbers on two die such that their sum is 6 and 2
    appears atleast on one die
19
20
21 PM=2/5 //since E has 5 elements and B has 2
    elements
22
23
24
25
26
27 //EXAMPLE 7.5(b)
28
29 disp("A couple has two children");
30
31
32 b=1; //boy child
33
34 g=2; //girl child
35
36 S=[11,12,21,22] ; //sample space where 11 implies
    both children being boys,12 implies first child
    being a boy and the second child being a girl
    and so on
37
38 disp("To find: probability(PM) that both children
    are boys ");
39
40
41

```

```

42 //7.5(b).i
43
44 L=S(:,1:3) //reduced sample space if it is known
    that one of the children is a boy
45
46
47 PM=1/length(L)
48
49
50 //7.5(b).ii
51
52 R=S(:,1:2) //reduced sample space if it is known
    that the older child is a boy
53
54
55 PM=1/length(R)

```

Scilab code Exa 7.6 Multiplication theorem for conditional probability

```

1 disp("A bag contains 12 items of which four are
    defective.Three items are drawn at random,one
    after the other");
2 s=12; //total itmes in the bag
3 d=4; //defective items in the bag
4 Pf=(s-d)/s ; //probability that the first item
    drawn is non defective
5 Pe=Pf*[(s-d-1)/(s-1)]*[(s-d-2)/(s-2)];
6 disp(Pe,'probability that all three items are non
    defective')
7 //after the first item is chosen,the second item is
    to be chosen from 1 less than the original number
    of items in the box and similarly the number of
    non defective items gets decreased by 1.Similarly
    ,for the third draw of item from the box

```

Scilab code Exa 7.7 Independent events

```
1 H=1;    //heads of a coin
2 T=2;    //tails of the coin
3 S=[111,112,121,122,211,212,221,222] //sample space
      for the toss of a coin three times. 111 implies
      heads all three times,112 implies heads on first
      two tosses and tails on the third toss
4 A=[111,112,121,122]; //event that first toss is
      heads
5 B=[111,112,211,212]; //event that second toss is
      heads
6 C=[112,211];          //event that exactly two heads
      appear in a row
7 PA=length(A)/length(S);
8 disp(PA,'probability of A is ')
9 PB=length(B)/length(S);
10 disp(PB,'probability of B is ')
11 PC=length(C)/length(S);
12 disp(PC,'probability of C is ')
13 AnB=intersect(A,B)
14 AnC=intersect(A,C)
15 BnC=intersect(B,C)
16 PAnB= length(AnB)/length(S);
17 disp(PAnB,'probability of the event AnB')
18 PAnC= length(AnC)/length(S);
19 disp(PAnC,'probability of the event AnC')
20 PBnC= length(BnC)/length(S);
21 disp(PBnC,'probability of the event BnC')
22 if((PA*PB)==PAnB),
23   disp("A and B are independent")
24 else
25   disp("A and B are dependent")
26 end
```



```

27 if((PA*PC)==PAnC),
28     disp("A and C are independent")
29 else
30     disp("A and C are dependent")
31 end
32 if((PB*PC)==PBnC),
33     disp("B and C are independent")
34 else
35     disp("B and C are dependent")
36 end

```

Scilab code Exa 7.8 Independent events

```

1 disp("Experiment: A and B both shoot at a target")
2 PA=1/4; //given probability of A hitting the target
3 PB=2/5; //given probability of B hitting the target
4 disp("A and B are independent events so PA*PB will
      be equal to probability of the event of A and B
      both hitting the target i.e PAnB")
5 PAnB=PA*PB;
6 PAorB=PA+PB-PAnB;
7 disp(PAorB, 'probability of atleast one of them
      hitting the target')

```

Scilab code Exa 7.9 Independent repeated trials

```

1 disp("Experiment: Three horses race together twice")
2 Ph1=1/2; //probability of first horse winning the
      race
3 Ph2=1/3; //probability of second horse winning the
      race
4 Ph3=1/6; //probability of third horse winning the
      race

```

```

5 S=[11,12,13,21,22,23,31,32,33] //sample space where
    11 implies first horse winning the first and
    second race both,12 implies first horse winning
    the first race and second horse winning the
    second race and so on
6 P11=Ph1*Ph1 //probability of first horse winning
    both races
7 P12=Ph1*Ph2 //probability of first horse winning
    the first race and second horse winning the
    second race
8 P13=Ph1*Ph3 //probability of first horse winning
    the first race and third horse winning the second
    race
9 P21=Ph2*Ph1 //probability of second horse winning
    the first race and first horse winning the second
    race
10 P22=Ph2*Ph2 //probability of second horse winning
    both the races
11 P23=Ph2*Ph3 //probability of second horse winning
    the first race and third horse winning the second
    race
12 P31=Ph3*Ph1 //probability of third horse winning
    the first race and first horse winning the second
    race
13 P32=Ph3*Ph2 //probability of third horse winning
    the first race and second horse winning the
    second race
14 P33=Ph3*Ph3 //probability of third horse winning
    both the races
15 disp(P31,'probability of third horse winning the
    first race and first horse winning the second
    race is ')

```

Scilab code Exa 7.10 Repeated trials with two outcomes

```

1 n=6; //number of times a fair coin is tossed and
    getting a heads is a success
2 p=1/2; //probability of getting a heads
3 q=1/2 ; //probability of not getting a heads
4 P2=(factorial(6)/(factorial(6-2)*factorial(2)))*p^2*
    q^(6-2);
5 disp(P2,'probability of getting exactly two heads (i
    .e k=2)')
6
7 P4=(factorial(6)/(factorial(6-4)*factorial(4)))*p^4*
    q^(6-4); //probabilty of getting four heads
8 P5=(factorial(6)/(factorial(6-5)*factorial(5)))*p^5*
    q^(6-5); //probabilty of getting five heads
9 P6=(factorial(6)/(factorial(6-6)*factorial(6)))*p^6*
    q^(6-6); //probabilty of getting five heads
10 PA=P4+P5+P6 ;
11 disp(PA,'probability of getting atleast four heads(i
    .e k=4,5 or 6)')
12
13 Pn=q^6 //probability of getting no heads
14 Pm=1-Pn;
15 disp(Pm,'probability of getting one or more heads')

```

Scilab code Exa 7.12 Random variables

```

1 disp("A box contains 12 items of which three are
    defective")
2 disp("A sample of three items is selected from the
    box")
3 s=factorial(12)/(factorial(12-3)*factorial(3));
4 disp(s,'number of elements in the sample space where
    samples are of size 3')
5 //X denotes the number of defective items in the
    sample
6 x=[0,1,2,3]; //range space of the random variable X

```

Scilab code Exa 7.13 Probability distribution of a random variable

```
1 r=[1,2,3,4,5,6,5,4,3,2,1];
2 //number of outcomes whose sum is
   2,3,4,5,6,7,8,9,10,11,12 respectively such that
   there is only 1 outcome i.e (1,1) whose sum is 2,
   two outcomes (1,2) and (2,1) whose sum is 3 and
   so on
3 t=36; //total number of
   elements in the sample space of the experiment of
   tossing a pair of dice
4 for i=1:11;
5 p=r(i)/t;
6 disp(p)
7 end
8 0.0277778 //probability of getting a sum
   of 2
9 0.0555556 //probability of getting a
   sum of 3
10 0.0833333 //probability of getting a sum
   of 4
11 0.1111111 //probability of getting a sum
   of 5
12 0.1388889 //probability of getting a sum
   of 6
13 0.1666667 //probability of getting a sum
   of 7
14 0.1388889 //probability of getting a sum
   of 8
15 0.1111111 //probability of getting a sum
   of 9
16 0.0833333 //probability of getting a sum
   of 10
17 0.0555556 //probability of getting a sum
```

```

    of 11
18 0.0277778           //probability of getting a sum
    of 12
19 x=[2,3,4,5,6,7,8,9,10,11,12];           //range space of
    random variable X which assigns to each point in
    sample space the sum of the numbers
20 D=[ 2,3,4,5,6,7,8,9,10,11,12;  0.0277778, 0.0555556
    , 0.0833333,  0.1111111, 0.1388889 ,0.1666667,
    0.1388889 ,0.1111111, 0.0833333, 0.0555556,
    0.0277778];
21 disp(D,'distribution table of X where first row
    gives the range space and second row gives the
    respective probabilities is as follows')

```

Scilab code Exa 7.14 Probability distribution of a random variable

```

1  disp("a box contains 12 items of which three are
    defective")
2  disp("A sample of three items is selected from the
    box")
3  r=factorial(9)/(factorial(9-3)*factorial(3)) //
    number of samples of size 3 with no defective
    items
4  t=220; //
    number of different samples of size 3 i.e the
    number of elements in the sample space
5  P0=r/t //
    probability of getting no defective item
6  r1=3*(factorial(9)/(factorial(9-2)*factorial(2)))
    //number of samples of size 3 getting 1
    defective item
7  P1=r1/t

    //probability of getting 1 defective item
8  r2=9*(factorial(3)/(factorial(3-2)*factorial(2)))

```

```

//number of samples of size 3 getting 2
defective item
9 P2=r2/t
//
probability of getting 2 defective item
10 r3=1; //number of samples of
size 3 getting 3 defective item
11 P3=r3/t //probability of getting 3
defective item
12 x=[0,1,2,3];
13 p=[P0,P1,P2,P3];
14 D=[0,1,2,3;P0,P1,P2,P3];
15 disp(D,'distribution table for random variable X the
upper row being values of X')

```

Scilab code Exa 7.15 Expectation of a random variable

```

1 disp("A fair coin is tossed six times");
2 x=[0,1,2,3,4,5,6]; //number of heads which can
occur
3 p=[1/64,6/64,15/64,20/64,15/64,6/64,1/64]; //
probability of occurring of heads where 1/64 is
probability for occurrence of a single head,6/64
that of occurrence of two heads and so on.
4 r=0;
5 for i=1:7;
6 r = r + (x(i)*p(i));
7 end
8 disp(r,'mean or expected number of heads are ')
9
10 disp("X is a random variable which gives possible
number of defective items in a sample of size 3")
;
11 //Box contains 12 items of which three are defective
12 x=[0,1,2,3]; //possible number of defective items

```

```

    in a sample of size 3
13 p=[84/220,108/220,27/220,1/220]; //probability of
    occurrence of each number in x respectively where
    84/220 is the probability for getting no
    defective item,108/220 is that of getting 1
    defective item and so on.
14 r=0;
15 for i=1:4;
16 r = r + (x(i)*p(i));
17 end
18 disp(r,'expected number of defective items in a
    sample of size 3 are')
19
20 Ph1=1/2; //probability of winning the race by
    first horse
21 Ph2=1/3; //probability of winning the race by
    second horse
22 Ph3=1/6; //probability of winning the race by
    third horse
23 //X is the payoff function for the winning horse
24 X1=2; //X pays $2 as first horse wins the race
25 X2=6; //X pays $6 as second horse wins the race
26 X3=9; //X pays $9 as third horse wins the race
27 E=X1*Ph1+X2*Ph2+X3*Ph3;
28 disp(E,'expected pay off for the race is')

```

Scilab code Exa 7.16 Variance and standard deviation of a random variable

```

1 u=3; //mean of distribution of random
    variable X
2 x=[0,1,2,3,4,5,6]; //values of X in the
    distribution as x where it is the number of times
    heads occurs when a coin is tossed six times
3 p=[1/64,6/64,15/64,20/64,15/64,6/64,1/64]; //

```

```

    probabilities of occurrence of each value of X (x
    ) in the distribution such that 1/64 gives the
    probability of occurrence of no heads at all ,6/64
    gives that of occurrence of heads for only one
    time and so on
4 k=0;
5 for i=1:7;
6 k=k+((x(i)-u)^2)*p(i);
7 end
8 disp(k, 'Variance of X is ')
9 s=sqrt(k);
10 disp(s, 'Standard deviation of X is ')
11
12 u=0.75; //mean
13 x=[0,1,2,3]; //values of random variable X as x in
    the probability distribution of X
14 p=[84/220,108/220,27/220,1/220]; //probability of
    values in x which appear in distribution table of
    X
15 g=0;
16 for i=1:4;
17 g=g+((x(i))^2)*p(i);
18 end
19 h=g-(u*u);
20 disp(h, 'variance of X is ')
21 sd=sqrt(h);
22 disp(sd, 'Standard deviation for X')

```

Scilab code Exa 7.17 Binomial diatribution

```

1 p=1/5; //probability of the man hitting a target
2 q=1-1/5; //probability of the man not hitting the
    target
3 n=100; //number of times the man fires
4 e=n*p;

```



```

5 disp(e, 'expected number of times the man will hit
   the target ')
6 r=sqrt(n*p*q);
7 disp(r, 'Standard deviation ')
8
9 p=1/2; //probability of guessing the correct answer
   in a five question true-false exam
10 n=5; //number of questions in the exam
11 g=n*p;
12 disp(g, 'expected number of correct answers in the
   exam ')

```

Scilab code Exa 7.18 Chebyshev inequality

```

1 u=75; //mean of a random variable X
2 n=5; //standard deviation of X
3 k=2; //for k=2
4 l1=u-k*n
5 l2=u+k*n
6 P1=1-(1/k)^2
7 disp("thus the probability that a value of X lies
   between 65 and 85 is atleast 0.75 according to
   Chebyshev inequality")
8 k=3; //for k=3
9 l1=u-k*n
10 l2=u+k*n
11 P2=1-(1/k)^2
12 disp("thus the probability that a value of X lies
   between 60 and 90 is atleast 0.8888889 according
   to Chebyshev Inequality")

```

Scilab code Exa 7.19 Sample mean and Law of large numbers

```
1 disp(" a die is tossed 5 times with the following
      outcomes")
2 x1=3;
3 x2=4;
4 x3=6;
5 x4=1;
6 x5=4;
7 xmean=(x1+x2+x3+x4+x5)/5 //mean of the outcomes
8 disp('for a fair die the mean is 3.5. So law of large
      numbers tells us that as number of outcomes
      increase for this experiment, there is a greater
      likelihood that the mean will get closer to 3.5')
```

Chapter 8

Graph Theory

Scilab code Exa 8.1 Paths and connectivity

```
1 // refer to page 8.6
2 disp('given a graph with 6 nodes viz. node1,node2
   ....node6 ')
3 A=[0 1 0 1 1 0;1 0 1 0 1 0;0 1 0 0 0 1;1 0 0 0 0 0;1
   1 0 0 0 0;0 0 1 0 0 0];
4 disp(A,'The adjacency matrix for A is ')
5 disp('sequence A is a path from node4 to node6; but
   it is not a trail since the edge from node1 to
   node2 is used twice')
6 B=[0 0 0 1 1 0;0 0 0 0 1 1;0 0 0 0 0 0;1 0 0 0 0 0;1
   1 0 0 0 0;0 1 0 0 0 0];
7 disp(B,'The adjacency matrix for B is ')
8 disp('sequence B is not a path since there is no
   edge from node2 to node6 is used twice')
9 C=[0 0 0 1 1 0;0 0 1 0 1 0;0 1 0 0 1 0;1 0 0 0 0 0;1
   1 1 0 0 1;0 0 0 0 1 0];
10 disp(C,'The adjacency matrix for C is ')
11 disp('sequence C is a trail since is no edge is used
   twice')
12 D=[0 0 0 1 1 0;0 0 0 0 0 0;0 0 0 0 1 1;1 0 0 0 0 0;1
   0 1 0 0 0;0 0 1 0 0 0];
```

```

13 disp(D, 'The adjacency matrix for D is ')
14 disp('sequence D is a simple path from node4 to
      node6 ')

```

Scilab code Exa 8.2 Minimum spanning tree

```

1  disp('to find: minimal spanning tree ')
2  disp('the adjacency matrix for the weighted graph(
      nodeA ,nodeB ... nodeF) of 6 nodes is :')
3  K=[0 0 7 0 4 7;0 0 8 3 7 5;7 8 0 0 6 0;0 3 0 0 0 4;4
      7 6 0 0 0;7 5 0 4 0 0]
4  disp('edges of the graph')
5  AC=7;
6  AE=4;
7  AF=7;
8  BC=8;
9  BD=3;
10 BE=7;
11 BF=5;
12 CE=6;
13 DF=4;
14 M=[AC ,AE ,AF ,BC ,BD ,BE ,BF ,CE ,DF]; //set of all edges
15 V=int32(M);
16 L=gsort(V) //edges sorted in decreasing order of
      their weights
17 disp('deleting edges without disconnecting the graph
      until 5 edges remain ')
18 N=[BE ,CE ,AE ,DF ,BD]; //edges in minimum spanning
      tree
19 Sum=sum(N);
20 disp(Sum, 'weight of the minimal spanning tree is ')
21
22
23 disp('another method of finding a minimal spanning
      tree is :')

```

```
24 K=gsort(V, 'g', 'i') //edges sorted in increasing
    order
25 N2=[BD,AE,DF,CE,AF]; //edges in minimum spanning
    tree
26 Sum2=sum(N2);
27 disp(Sum2, 'weight of the minimal spanning tree is')
```

Chapter 9

Directed graphs

Scilab code Exa 9.6 Adjacency matrix

```
1 A=[0 0 0 1;1 0 1 1;1 0 0 1;1 0 1 0];
2 disp(A,'adjacency matrix of graph G is ')
3 A2=A^2
4 A3=A^3
5 disp('the number of ones in A is equal to the number
      of edges in the graph i.e 8')
```

Scilab code Exa 9.8 Path matrix

```
1 A=[0 0 0 1;1 0 1 1;1 0 0 1;1 0 1 0];
2 disp(A,'adjacency matrix of graph G is ')
3 A4=A^4;
4 A3=A^3;
5 A2=A^2;
6 B4=A+A2+A3+A4;
7 B4=[4 11 7 7 0 0 0 0 3 7 4 4 4 11 7 7];
8 for i=1:16
9 if(B4(i)~=0) then
```

```
10 B4(i)=1;
11 end
12 end
13 disp(B4,'Replacing non zero entries of B4 with 1 ,we
    get path (reachability) matrix P is:')
14 disp('there are zero entries in P,therefore the
    graph is not strongly connected')
```

Chapter 11

Properties of the integers

Scilab code Exa 11.2 Division algorithm

```
1 disp('Division Algorithm ')
2 a=4461; //dividend
3 b=16; //divisor
4 r=modulo(a,b) //remainder
5 k=fix(a/b) //quotient
6 j=b*k+r //dividend=divisor*quotient+remainder
7
8 a=-262; //dividend
9 b=3; //divisor
10 k=fix(a/b) //remainder
11 r=modulo(a,b) //quotient
12 j=b*k+r //dividend=divisor*quotient+remainder
13 disp('a and j have equal values.Hence division
algorithm is proved')
```

Scilab code Exa 11.4 Primes

```
1 disp('Divisibility and Primes')
```



```

2 x=50;
3 disp('prime numbers less than 50 are')
4 y=primes(x)
5
6 disp('the prime factorisation of 21,24 and 1729
       respectively are:')
7 k=factor(21)
8 l=factor(24)
9 n=factor(1729)

```

Scilab code Exa 11.5 Greatest Common Divisor

```

1 disp('the GCD of the following numbers is:')
2 V=int32([12,18]);
3 [thegcd]=gcd(V)
4 V=int32([12,-18]);
5 [thegcd]=gcd(V)
6 V=int32([12,-16]);
7 [thegcd]=gcd(V)
8 V=int32([29,15]);
9 [thegcd]=gcd(V)
10 V=int32([14,49]);
11 [thegcd]=gcd(V)

```

Scilab code Exa 11.6 Euclidean algorithm

```

1 disp('Euclidean Algorithm')
2 a=[540,168,36,24];
3 b=[168,36,24,12];
4 for i=1:4
5 V=int32([a(i),b(i)]);
6 thegcd=[];
7 thegcd(i)=gcd(V);

```

```

8  disp(thegcd(i))
9  end
10
11 function []=myf(dividend,divisor)
12 quotient=floor(dividend/divisor);
13 rem=modulo(dividend,divisor);
14 k=quotient*divisor+rem;
15 disp(k)
16 if(rem~=0) then
17     myf(divisor,rem)
18 end
19 endfunction
20
21 myf(540,168)
22
23 disp('for the equation 540*x+168*y=12,we are given ')
24 a=540;
25 b=168;
26 c=24;
27 d=36;
28 d=a-3*b;      //Eqn (1)
29 c=b-4*d;      //Eqn (2)
30 k=d-1*c;      //Eqn (3)
31 5*d-1*b;      //Eqn (4)
32 k=d-b+4*d;    //substituting value of c in Eqn (3)
    from Eqn (2)
33 r=5*a-16*b;
34 if(r==k) then
35     disp('x=5 and y=16');
36 end

```

Scilab code Exa 11.9 Fundamental theorem of Arithmetic

```

1  a=2^4*3^3*7*11*13
2  b=2^3*3^2*5^2*11*17

```

```

3 V=int32([a,b]);
4 [d]=gcd(V)
5 lcm1=2^4*3^3*5^2*7*11*13*17 //lcm is the product
  of those primes which appear in either a or b

```

Scilab code Exa 11.12 Congruence relation

```

1 x=poly(0,'x');
2 g=3*x^2-7*x+5
3 m=horner(g,2) //value of polynomial at 2
4 n=horner(g,8) //value of polynomial at 8
5 j=m-n
6 disp(n,"for n = ")
7 if(modulo(j,6)==0) then
8 mprintf('%i is congruent to %i(mod 6)',m,n)
9 end

```

Scilab code Exa 11.19 Linear congruence equation

```

1 disp('solving for the congruence equation 8x @ 12(
  mod 28) ,where @ is the sign for congruence')
2 a=8;
3 b=12;
4 m=28;
5 V= int32([a,m]);
6 d= gcd(V);
7 a1= a/d;
8 b1= b/d;
9 m1= m/d;
10
11 function yd= f(x)
12 yd= (a1*x)-b1
13 endfunction

```

```
14
15  disp('k is the unique solution of the equation ')
16  for i=0 : m1
17  x=i;
18  p=f(x);
19  if(modulo(p,m1) == 0)
20  k=x
21  break;
22  end
23  end
24
25  s1=k;
26  s2=k+m1;
27  s3=k+(m1*2);
28  s4=k+(m1*3);
29  disp('solutions of the original equation at d=4')
30  disp(s1)
31  disp(s2)
32  disp(s3)
33  disp(s4)
```

Chapter 12

Algebraic Systems

Scilab code Exa 12.4 Properties of operations

```
1 a=(8-4)-3
2 b=8-(4-3)
3 disp('since a and b are not equal so subtraction is
      non-commutative on Z(set of integers)')
4
5 a=[1 2;3 4]
6 b=[5 6;0 -2]
7 g= a*b
8 k= b*a
9 disp('since g and k are not equal matrix
      multiplication is non-commutative')
10
11 h=(2^2)^3
12 j=2^(2^3)
13 disp('since h and j are not equal so exponential
      operation is non associative on the set of
      positive integers N')
```

Scilab code Exa 12.17 Roots of polynomial

```
1 t=poly(0, 't');
2 f=t^3+t^2-8*t+4
3 g=factors(f)
4 disp(r=roots(f), 'roots of f(t) are as follows:')
5
6 t=poly(0, 't');
7 h=t^4-2*t^3+11*t-10
8 disp(r=roots(h), 'the real roots of h(t) are 1 and -2
   ')
```

Scilab code Exa 12.18 Roots of polynomial

```
1 t=poly(0, 't');
2 f=t^4-3*t^3+6*t^2+25*t-39
3 g=factors(f)
4 disp(r=roots(f), 'roots of f(t) are as follows:')
```

Chapter 15

Boolean Algebra

Scilab code Exa 15.1 Basic definitions in boolean algebra

```
1 //0 denotes False and 1 denotes true
2 b=[0,1];
3 //binary operation + on the set of bits
4 for i=1:2
5 for j=1:2
6 k = b(i)& b(j);
7 disp(k)
8 end
9 end
10 //binary operation * on the set of bits
11 for i=1:2
12 for j=1:2
13 k = b(i)| b(j);
14 disp(k)
15 end
16 end
17 //unary operation ' on the set of bits
18 k=~b
19 clear;
20 D=[1,2,5,7,10,14,35,70];
21 a=35;
```

```
22 b=70;
23 V=int32([a,b]);
24 thelcm=lcm(V) //a+b=lcm(a,b)
25 V=int32([a,b])
26 thegcd=gcd(V) //a*b=gcd(a,b)
27 abar=70/a //a'=70/a
```

Scilab code Exa 15.2 Boolean algebra as lattices

```
1 D=[1,2,5,7,10,14,35,70];
2 a = 2; //a and b belong to D
3 b = 14;
4 V=int32([a,b]);
5 thelcm=lcm(V)
6 V=int32([a,b]);
7 thegcd=gcd(V)
8 abar=70/a
9 bbar=70/b
10 j=[abar,b];
11 h=[a,bbar];
12 V=int32([j])
13 lcm1=lcm(V)
14 K=int32([h])
15 lcm2=lcm(K)
```

Chapter 16

Recurrence relations

Scilab code Exa 16.14 Linear homogenous recurrence relations with constant coefficients

```
1 a=[];
2 a(1)=1; //initial condition
3 a(2)=2; //initial condition
4 disp('for recurrence relation  $a(n)=5*a(n-1)-4*a(n-2)+n^2$ ') //this is a second order recurrence
    relation with constant coefficients.It is non
    homogenous because of the  $n^2$ 
5 for n=3:4
6 a(n)=5*a(n-1)-4*a(n-2)+n^2;
7 mprintf('Value of a(%i) is: %i \n',n,a(n))
8 end
9
10 a=[];
11 a(1)=1; //initial condition
12 a(2)=2; //initial condition
13 disp('for recurrence relation  $a(n)=2*a(n-1)*a(n-2)+n^2$ ') //this recurrence relation is not linear
14 for n=3:4
15 a(n)=2*a(n-1)*a(n-2)+n^2;
16 mprintf('Value of a(%i) is: %i \n',n,a(n))
```

```

17 end
18
19 a=[];
20 a(1)=1;    //initial condition
21 a(2)=2;    //initial condition
22 disp('for recurrence relation a(n)=n*a(n-1)+3*a(n-2)
      ') //this is a homogenous linear second order
      recurrence relation without constant coefficients
      because the coefficient of a[n-1] is n,not a
      constant
23 for n=3:4
24 a(n)=n*a(n-1)+3*a(n-2);
25 mprintf('Value of a(%i) is: %i \n',n,a(n))
26 end
27
28
29 a=[];
30 a(1)=1;    //initial condition
31 a(2)=2;    //initial condition
32 a(3)=1;    //initial condition
33 disp('for recurrence relation a(n)=2*a(n-1)+5*a(n-2)
      -6*a(n-3)') //this is a homogenous linear third
      order recurrence relation with constant
      coefficients.Thus we need three ,not two ,initial
      conditions to yield a unique solution of the
      recurrence relation
34 for n=4:6
35 a(n)=2*a(n-1)+5*a(n-2)-6*a(n-3);
36 mprintf('Value of a(%i) is: %i \n',n,a(n))
37 end

```

Scilab code Exa 16.15 Solving linear homogenous recurrence relations with constant coefficients

```

1 disp('recurrence relation of Fibonacci numbers f[n]=

```

```

    f[n-1]+f[n-2]')
2  x=poly(0,'x');
3  g=x^2-x-1;
4  disp(g,'characteristic equation of the recurrence
    relation is:')
5  j=[];
6  j=roots(g);
7  disp(j,'roots of the characteristic equation j1,j2')
8  disp('for general equation fn=Ar^n+Br^n, values of
    Aand B respectively are calculated as:')
9  disp('initial condition at n=0 and n=1 respectively
    are:')
10 f1=1;
11 f2=1;
12 //putting the values of f1 and f2 we get the
    equations to solve
13 D=[ 1.6180340 -0.618034;(1.6180340)^2  (-0.618034)
    ^2];
14 K=[1 1]';
15 c=[];
16 c=D\K;
17 A=c(1)
18 B=c(2)
19
20 disp('thus the solution is f[n
    ]=0.4472136*((1.618034)^n-(- 0.4472136)^n)')

```

Scilab code Exa 16.16 Solving linear homogenous recurrence relations with constant coefficients

```

1  disp('The recurrence relation t[n]=3t[n-1]+4t[n-2]')
2  x=poly(0,'x');
3  g=x^2-3*x-4;
4  disp(g,'characteristic polynomial equation for the
    above recurrence relation')

```

```

5 j=[];
6 j=roots(g);
7 disp(j,'roots of the characterstic equation j1,j2')
8 disp('general solution t[n]=c1*(-1)^n+c2*4^n')
9 disp('initial condition at n=0 and n=1 respectively
      are:')
10 t0=0;
11 t1=5;
12 //putting the values of t0 and t1 we get the
      equations to solve
13 D=[1 1;-1 4]
14 K=[0 5]'
15 c=[];
16 c=D\K;
17 c1=c(1)
18 c2=c(2)
19 disp('thus the solution is t{n}=4^n-(-1)^n')

```

Scilab code Exa 16.17 Solving linear homogenous recurrence relations with constant coefficients

```

1 disp('The recurrence relation t[n]=4(t[n-1]-t[n-2])')
  )
2 x=poly(0,'x');
3 disp(g=x^2-4*x+4,'characterstic polynomial equation
      for the above recurrence relation')
4 j=[];
5 j=roots(g);
6 disp(j,'roots of the characterstic equation j1,j2')
7 disp('the general solution is t[n]=n*2^n)
8 disp('initial condition at n=0 and n=1 respectively
      are:')
9 t0=1;
10 t1=1;
11 //putting the values of t0 and t1 we get the

```

```

        equations to solve
12 D=[1 0;2 2]
13 K=[1 1] '
14 c=linsolve(D,K)
15 D=[1 0;2 2]
16 K=[1 1] '
17 c=[];
18 c=D\K;
19 c1=c(1)
20 c2=c(2)
21 disp('thus the solution is t{n}=2*n-n*2^(n-1)')

```

Scilab code Exa 16.18 Solving linear homogenous recurrence relations with constant coefficients

```

1 disp('The recurrence relation a[n]=2*a[n-1]-3a[n-2] '
    )
2 x=poly(0,'x');
3 disp(g=x^2-2*x-3,'characterstic polynomial equation
    for the above recurrence relation')
4 j=[];
5 j=roots(g);
6 disp(j,'roots of the characterstic equation j1,j2')
7 disp('the general solution is a[n]=c1*3^n+c2*(-1)^n'
    )
8 disp('initial condition at n=0 and n=1 respectively
    are:')
9 //putting the values of t0 and t1 we get the
    equations to solve
10 a0=1;
11 a1=2;
12 D=[1 1;3 -1]
13 K=[1 2] '
14 c=[];
15 c=D\K;

```

```

16 c1=c(1)
17 c2=c(2)
18 disp('thus the solution is a[n]=0.75*(3^n)+0.25*(1^n
      )')

```

Scilab code Exa 16.19 Solving general homogenous linear recurrence relations

```

1 disp('The recurrence relation a[n]=11*a[n-1]-39*a[n-2]+45*a[n-3]')
2 x=poly(0,'x');
3 disp(g=x^3-11*x^2+39*x-45,'characterstic polynomial
      equation for the above recurrence relation')
4 j=[];
5 j=roots(g);
6 disp(j,'roots of the characterstic equation j1,j2')
7 disp('hence the general solution is:a[n]=c1*(3^n)+c2
      *n*(3^n)+c3*(5^n)')
8 disp('initial condition at n=0 and n=1 respectively
      are:')
9 //putting the values of t0 and t1 we get the
      equations to solve
10 a0=5;
11 a1=11;
12 a2=25;
13 D=[1 0 1;3 3 5;9 18 25];
14 K=[5 11 25]'
15 c=[];
16 c=D\K;
17 c1=c(1)
18 c2=c(2)
19 c3=c(3)
20 disp('thus the solution is a[n]=(4-2*n)*(3^n)+5^n')

```
